INF721

2024/2

Deep Learning

L2: Machine Learning

Logistics

Announcements

- We have a **google spaces** for the course
- Lecture 1 is already available on the course webpage

Last Lecture

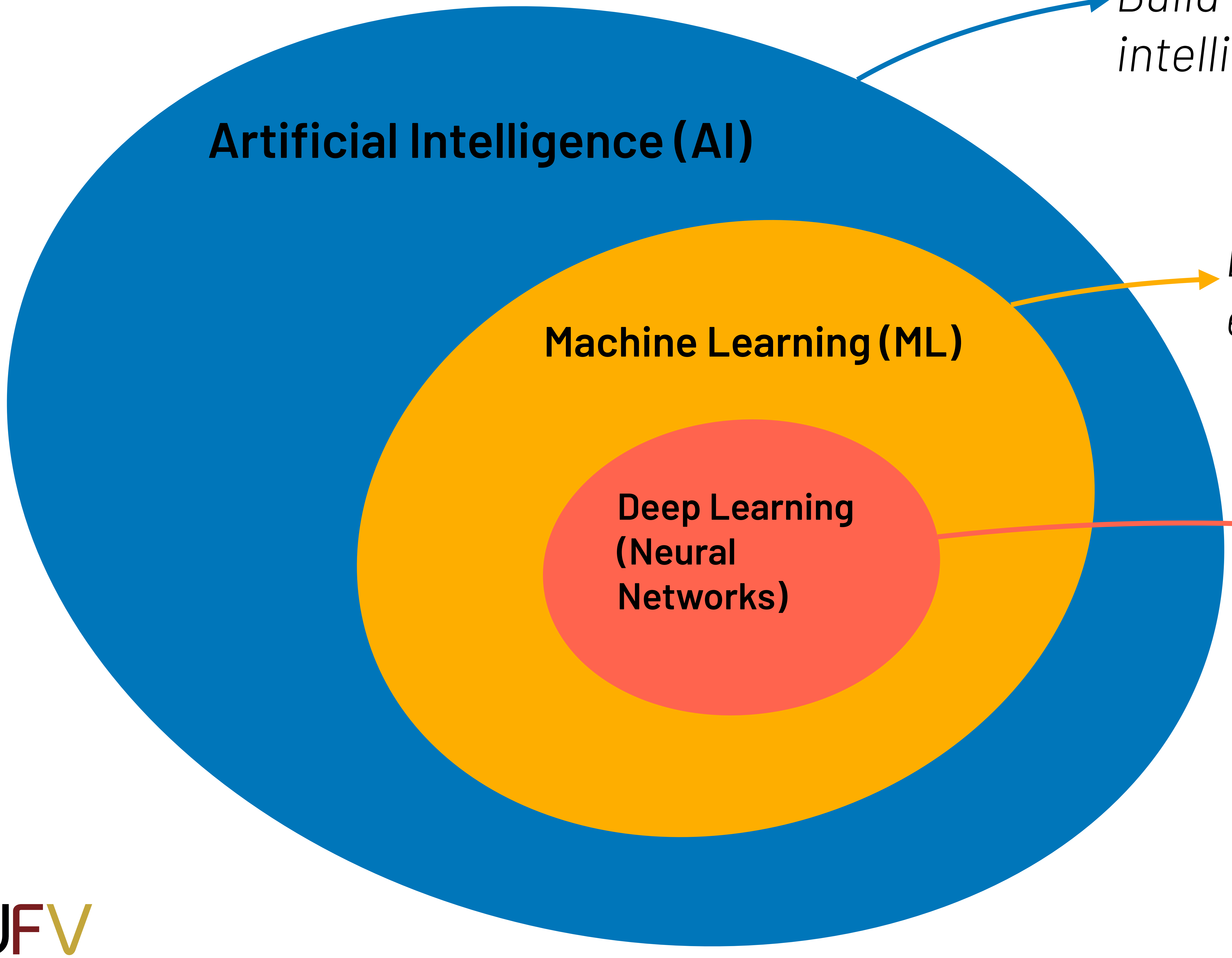
- Motivation
- Course syllabus

Lecture outline

- Machine Learning
  - Brief History
  - Fomulation
  - Types of problems
- Supervised Learning Algorithms
  - Hypothesis space
  - Loss function
  - Evaluating Performance

Machine Learning

Machine Learning

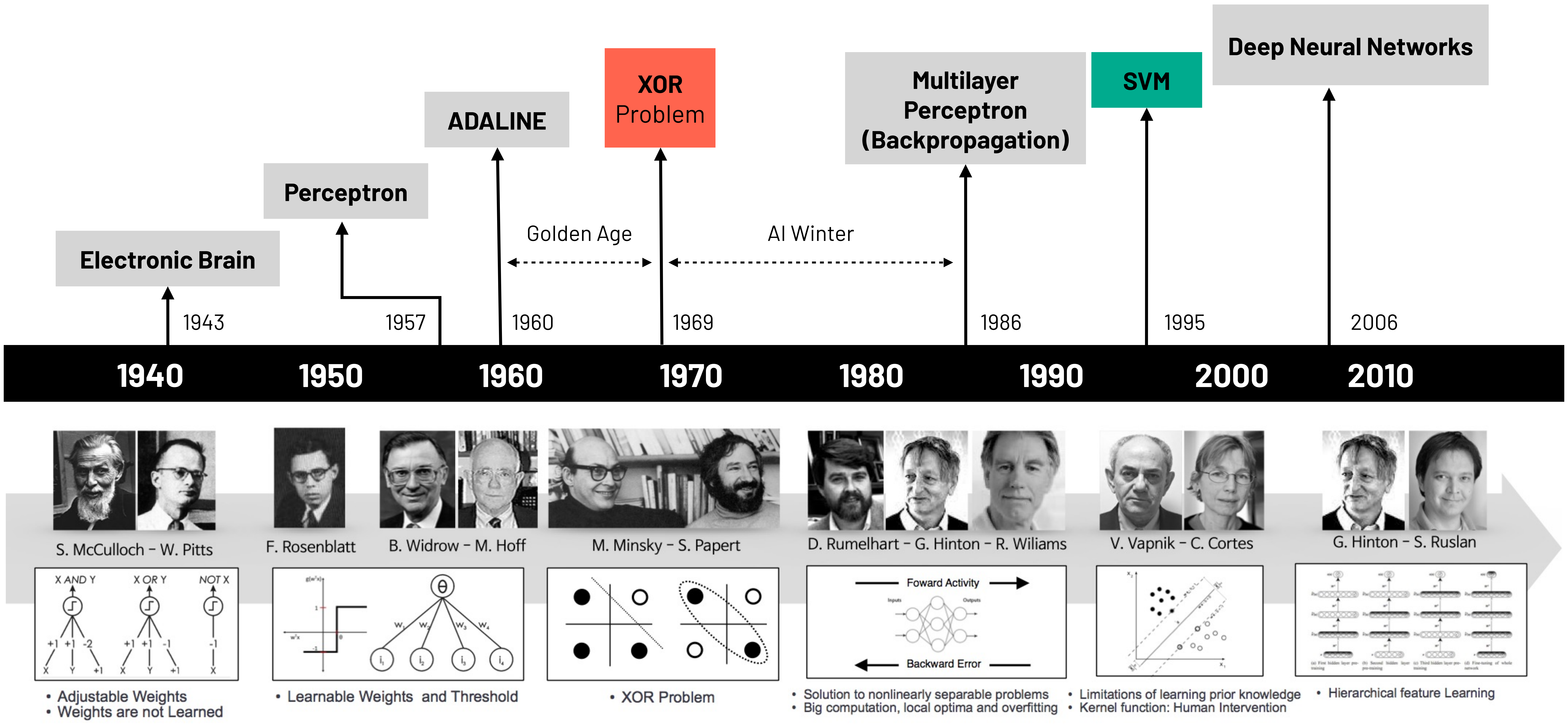


Build computer systems that can simulate human intelligent habilities (learning, reasoning,...)

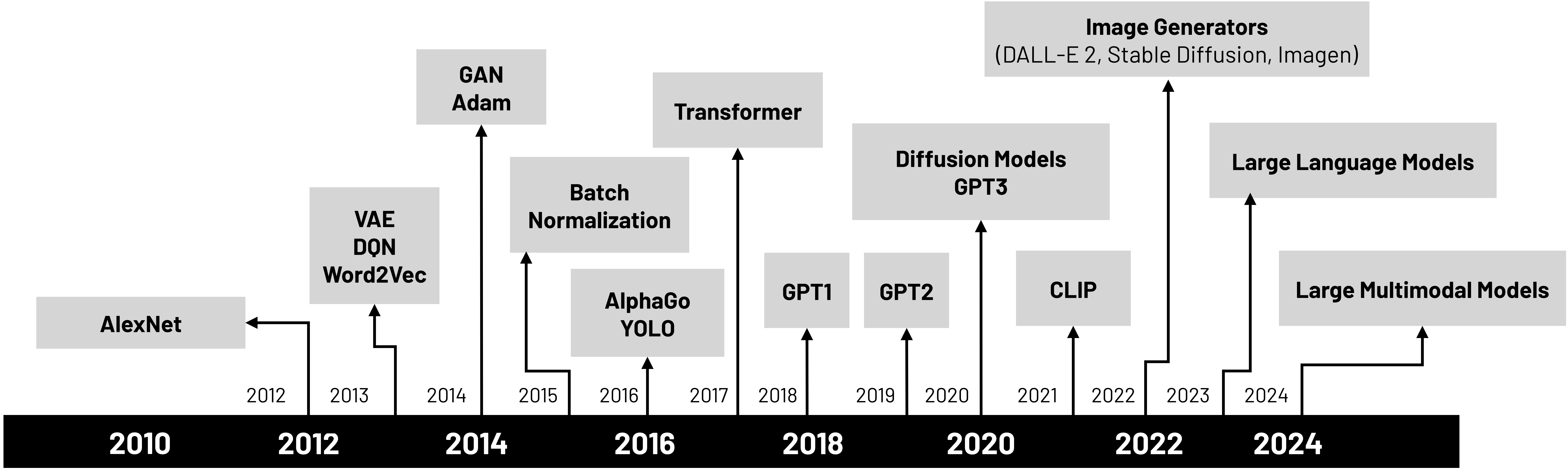
Learn to perform tasks from previous experiences (without explicit instructions)

Machine learning model inspired by the structure and function of biological neural networks in animal brains

Machine Learning



Machine Learning



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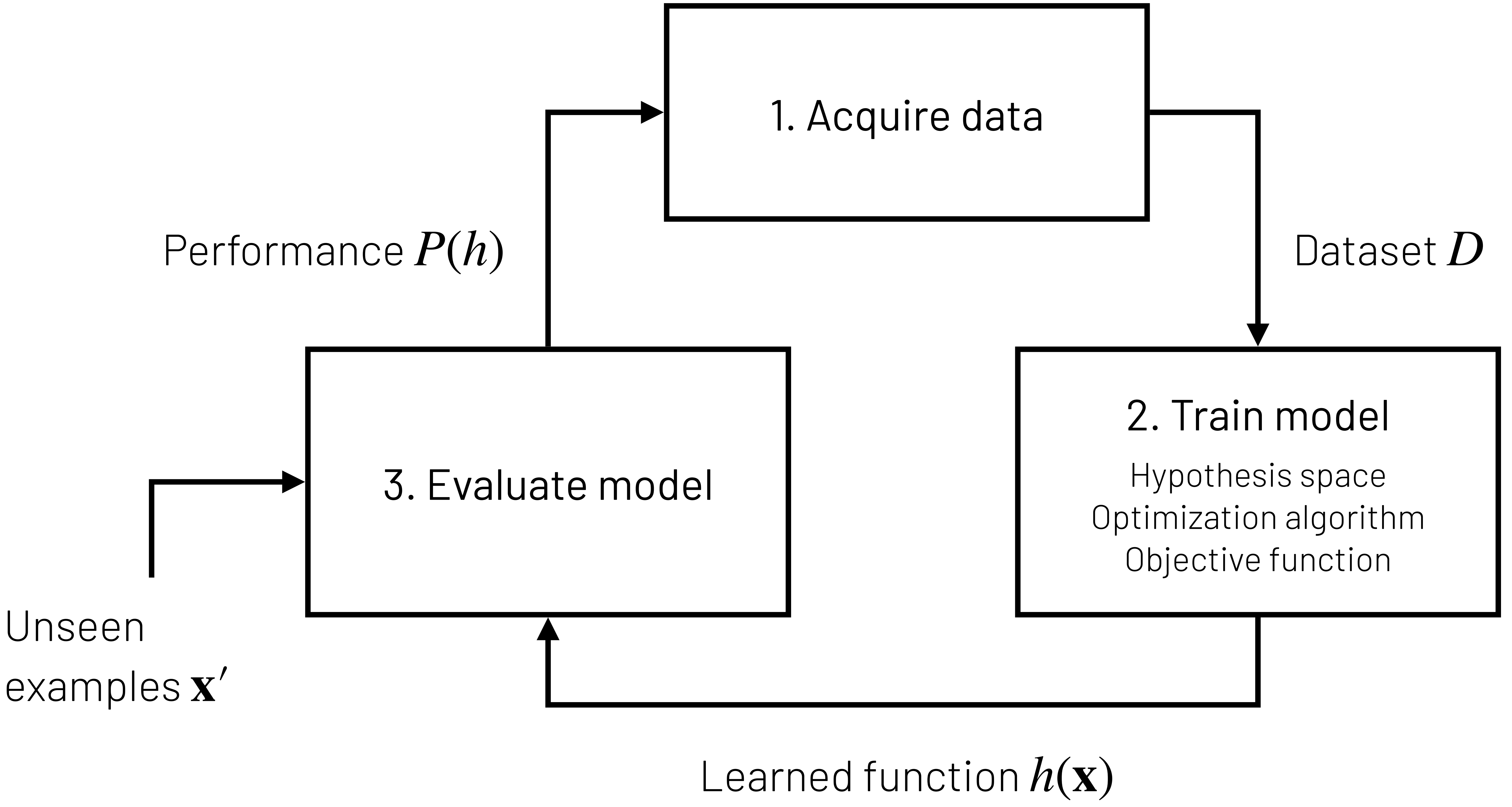
Machine Learning (ML)

Given a dataset D and a performance metric P, we want to learn a function $h(\mathbf{x})$ (i.e., model) that maximizes $P(h)$ on unseen examples $\mathbf{x}' \notin D$



The type of learning is defined by the type of experience given to the model:

Labelled Data

- **Supervised Learning**

Unlabelled Data

- **Unsupervised Learning**

Rewards from the environment

- **Reinforcement Learning**

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Supervised Learning

In **supervised learning** problems, we have a dataset D of tuples $(\mathbf{x}^{(i)}, y^{(i)})$ and the goal is to learn a function $h(\mathbf{x}) = \hat{y}$ that predict the labels $y^{(i)}$ from the feature vectors $\mathbf{x}^{(i)}$, minimizing predicion error on unseen examples $\mathbf{x}' \notin D$.

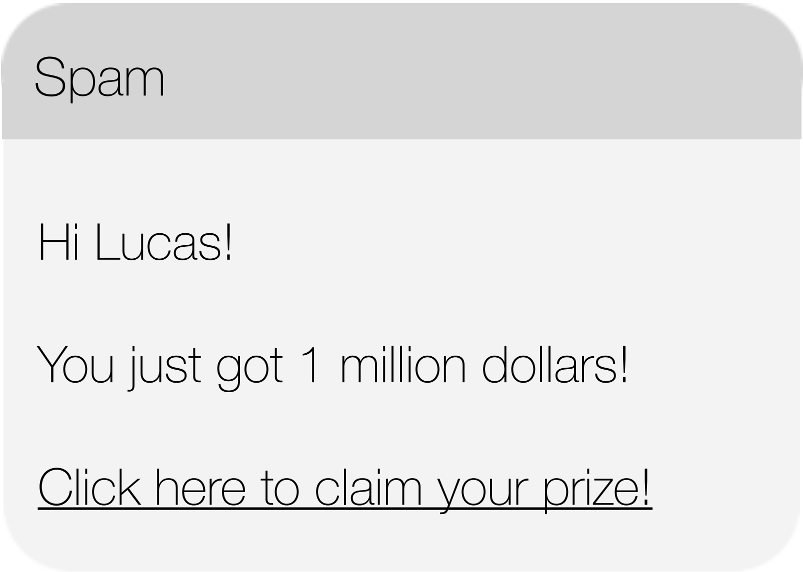
Formally:

$D = \{(\mathbf{x}^{(1)}, y^{(1)}), \ldots, (\mathbf{x}^{(m)}, y^{(m)})\} \subseteq \mathbb{R}^d \times C$, where:

- m is the number of examples in the dataset
- $\mathbf{x}^{(i)}$ is the feature vector of the i^{th} example
- $y^{(i)}$ is the lavel (or class) of the i^{th} example
- $\mathbb{R}^d$ is d-dimensional feature space
- C is the label space

Examples of supervised learning problems

$$D = \{(\mathbf{x}^{(1)}, y^{(1)}), \ldots, (\mathbf{x}^{(m)}, y^{(m)})\} \subseteq \mathbb{R}^d \times C$$



Spam Detection (Binary classification)

$\mathbf{x}^{(i)}$: frequency of the i^{th} word in a dictionary (bag of words)

$y^{(i)} \in C = \{0,1\}$

Handwritten Digits Classification (Multiclass classification)

$\mathbf{x}^{(i)}$: color value of the i^{th} pixel of the flattened image

$y^{(i)} \in C = \{0,1,2,3,4,5,6,7,8,9\}$

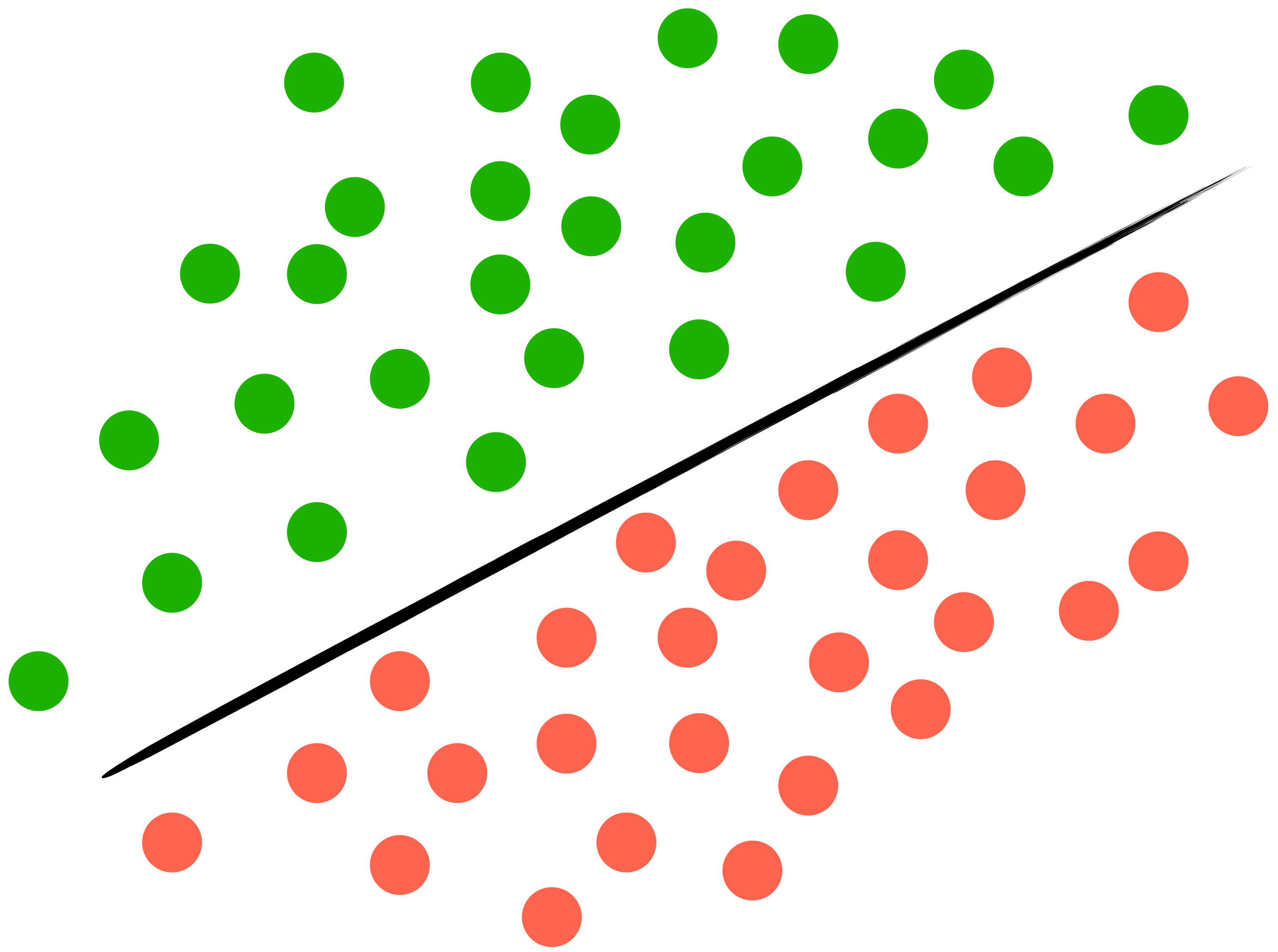
Houce price prediction (Regression)

$\mathbf{x}_1^{(i)}$: size , $\mathbf{x}_2^{(i)}$: location, ..., $\mathbf{x}_n^{(i)}$: number of bedrooms

$y^{(i)} \in C = \mathbb{R}$

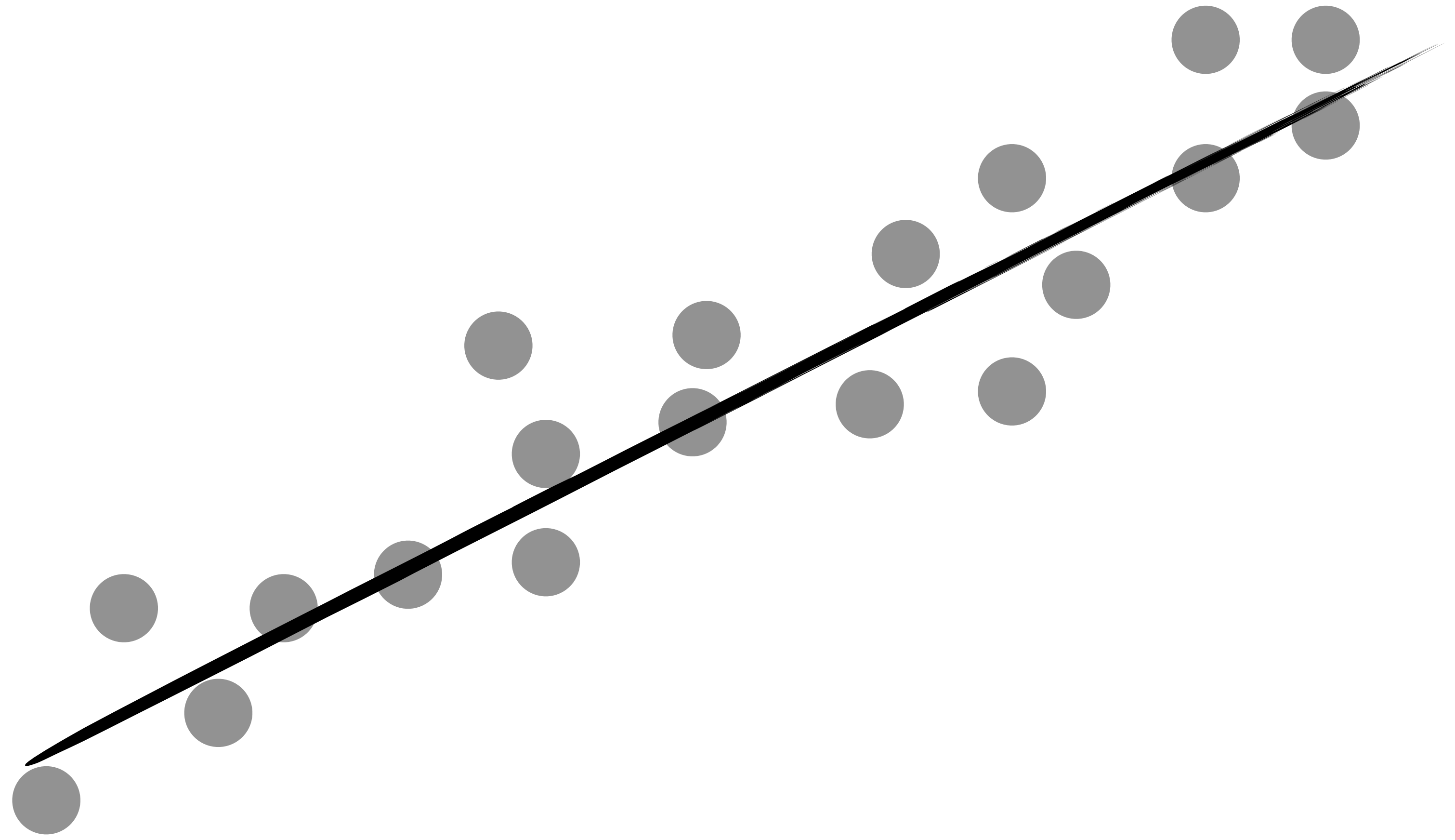
Visualization of Supervised Learning Problems

Classification



Find a function (e.g., linear) that splits the data points by their classes

Regression



Find a function (e.g., linear) that fits the data points

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Unsupervised Learning

In **unsupervised learning,** the examples in the dataset D do not have labels $y^{(i)}$ and the goal is to discover patterns and relationships in the data.

Formally:

$D = \{\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \cdots, \mathbf{x}^{(m)}\} \subseteq \mathbb{R}^d$, where:

- m is the number of examples in the dataset
- $\mathbf{x}^{(i)}$ is the feature vector of the i^{th} example
- $\mathbb{R}^d$ is d-dimensional feature space

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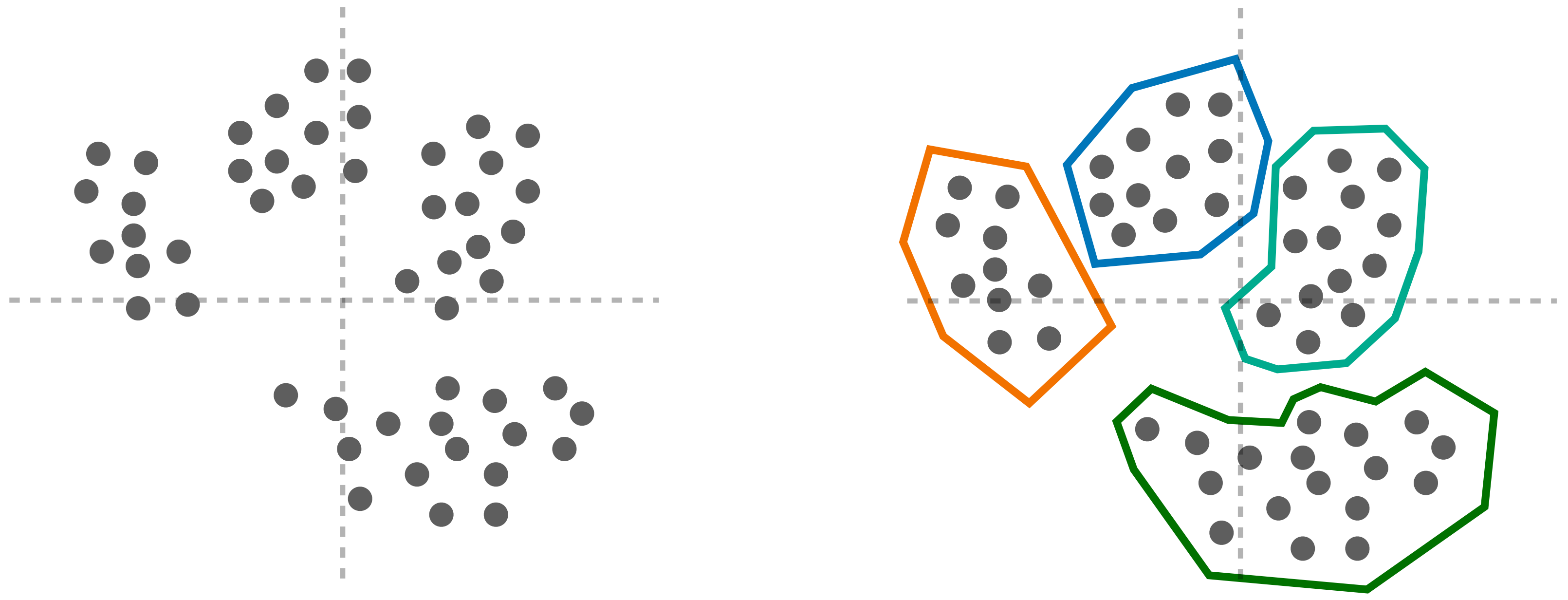
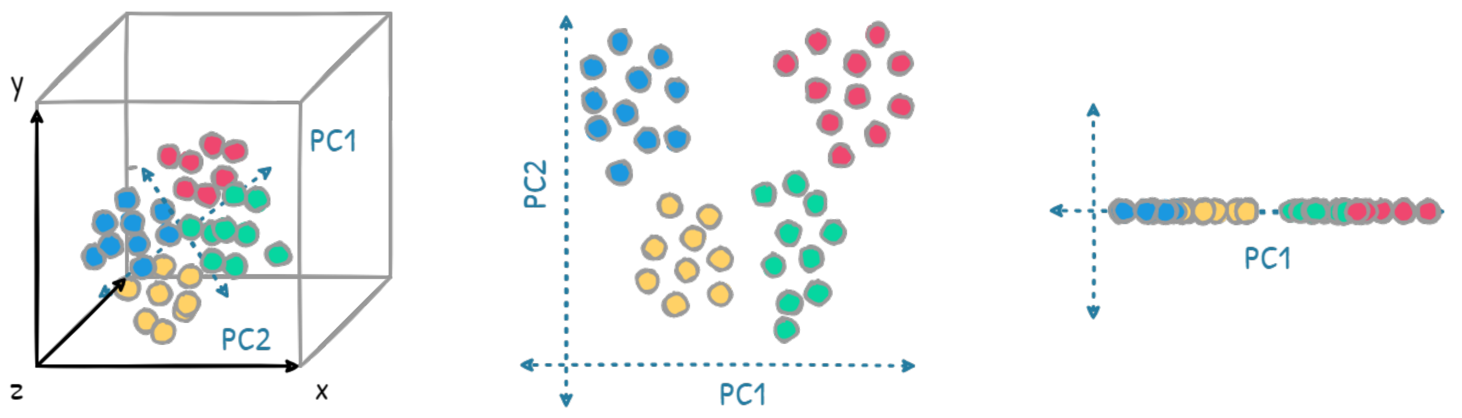
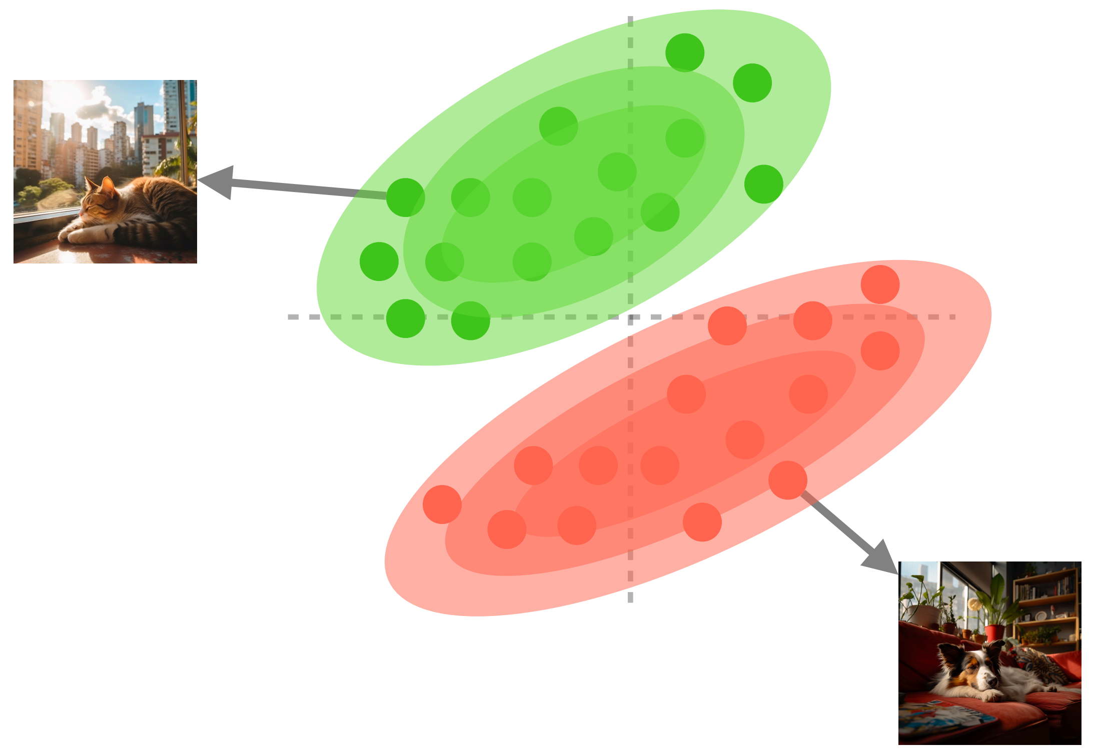
Examples of unsupervised learning problems



Clustering
Discover the inherent groupings in the data



Dimentionality Reduction
Representing a given dataset using a lower number of features



Generative AI
Generate new examples similar to the dataset

Reinforcement Learning

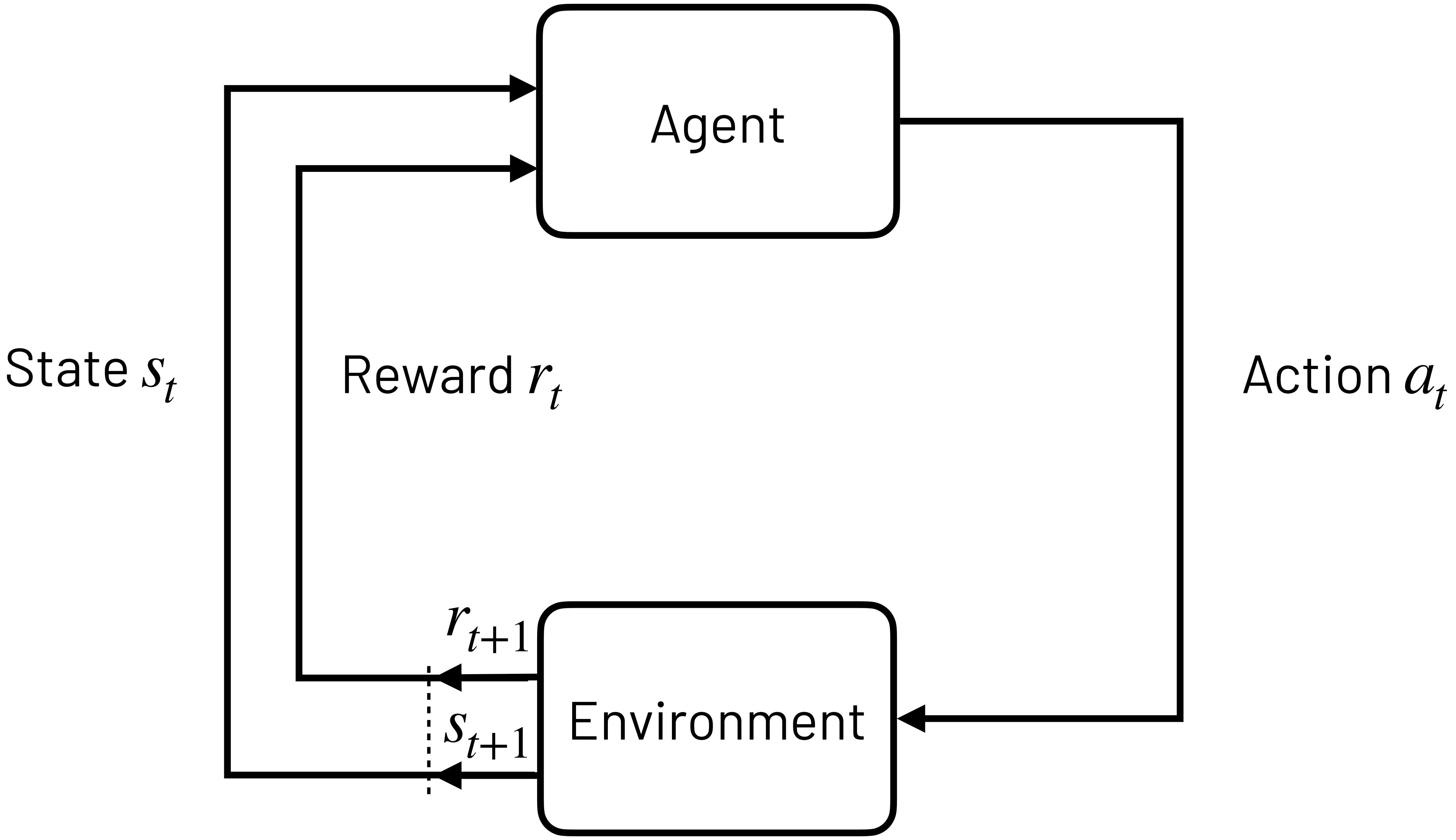
In reinforcement learning, the goal is to learn a function $\pi(s) = a$ to predict an action a from a state s, maximizing the expected sum of rewards received by the environment.

Agent

- Receives a state s_t at time t
- Performs an action a_t at time t

Environment

- Returns a reward value r_{t+1} and;
- The next state s_{t+1}



Data Types

Structured Data (tabular)

Size (m2)	Location	N. of bedrooms	...	Price
72	Centro	2		
54	Centro	1		
...	...	...		...
72	Clélia	3		

Age	State	Ad Id	...	Click
72	MG	93242		1
54	SP	93287		0
...	...	...		...
72	RJ	71244		1

Unstructured Data

Images

Spam

Hi Lucas!

You just got 1 million dollars!

Click here to claim your prize!

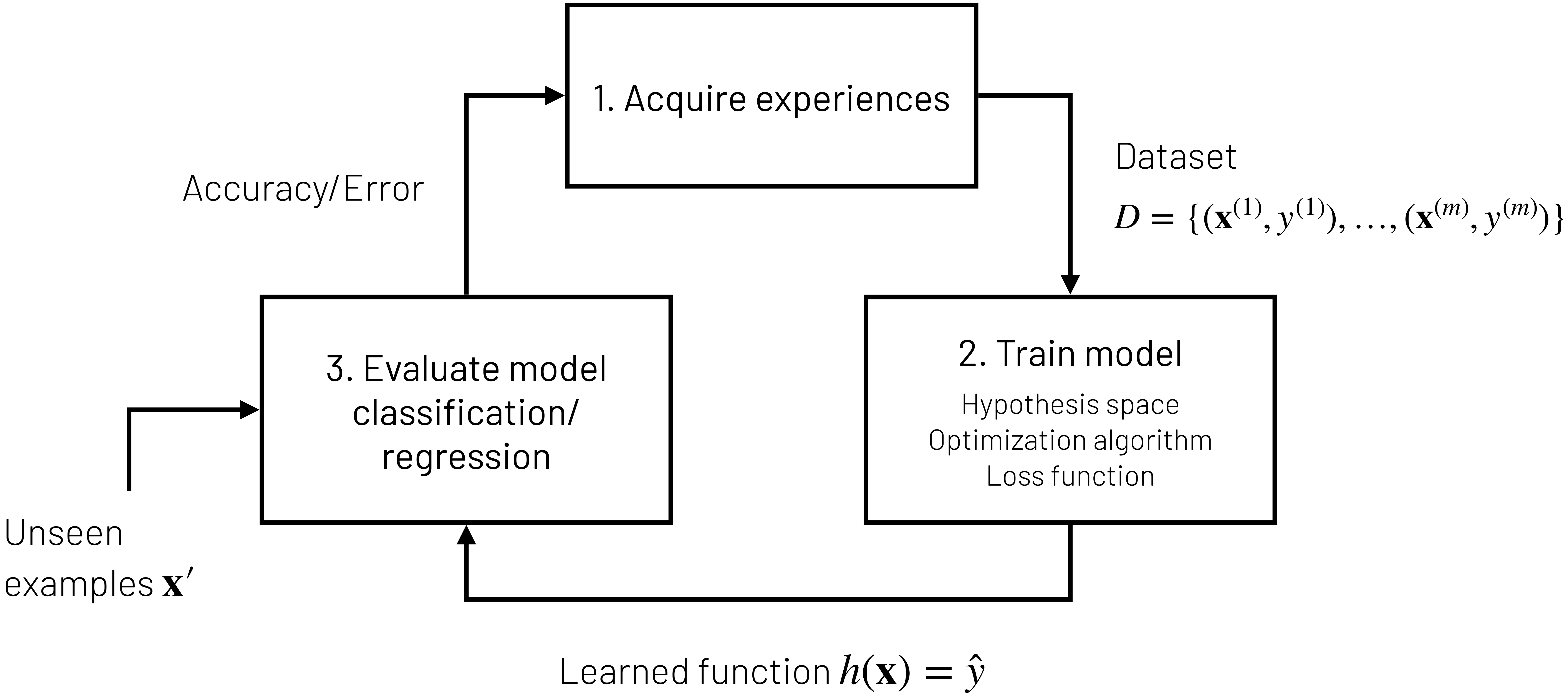
Text

Audio

Supervised Learning Algorithms

Supervised Learning

Learn a function $h(\mathbf{x}) = \hat{y}$ from a dataset $D = \{(\mathbf{x}^{(1)}, y^{(1)}), \ldots, (\mathbf{x}^{(m)}, y^{(m)})\}$ to predict the labels $y^{(i)}$ from the feature vectors $\mathbf{x}^{(i)}$, minimizing prediction error on unseen examples $\mathbf{x}'$



Training a model

Training a model means finding a function $h \in H$ in a space of functions H

To do that, a supervised learning algorithm needs to:

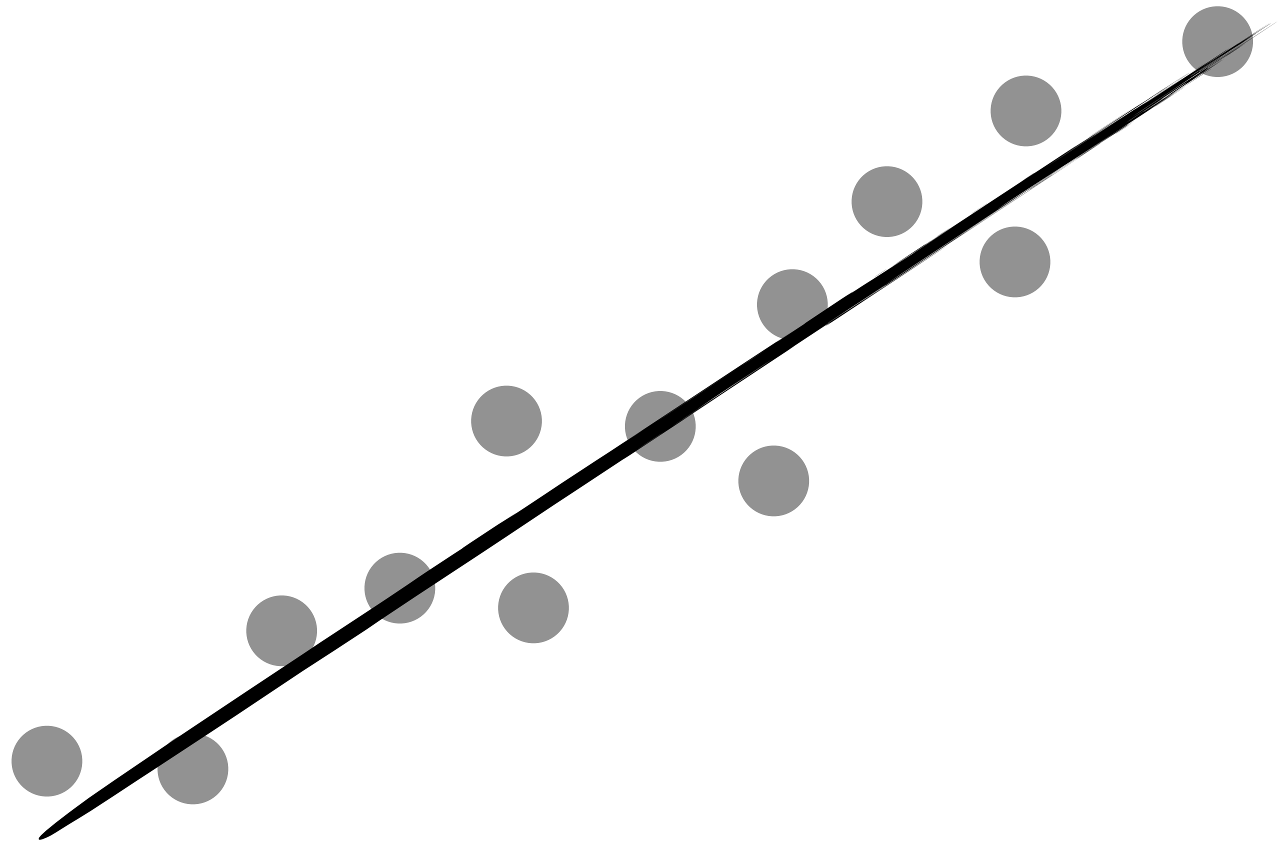
1. Define an especific espace of funcions, called **hypothesis space** H;
2. Find the best function $h \in H$, i.e., the function that minimizes the prediction error according to some **loss function** L.

   In neural networks (and many other ML algorithms), this step is formalized as an optimization algorithm!

Hypothesis Space

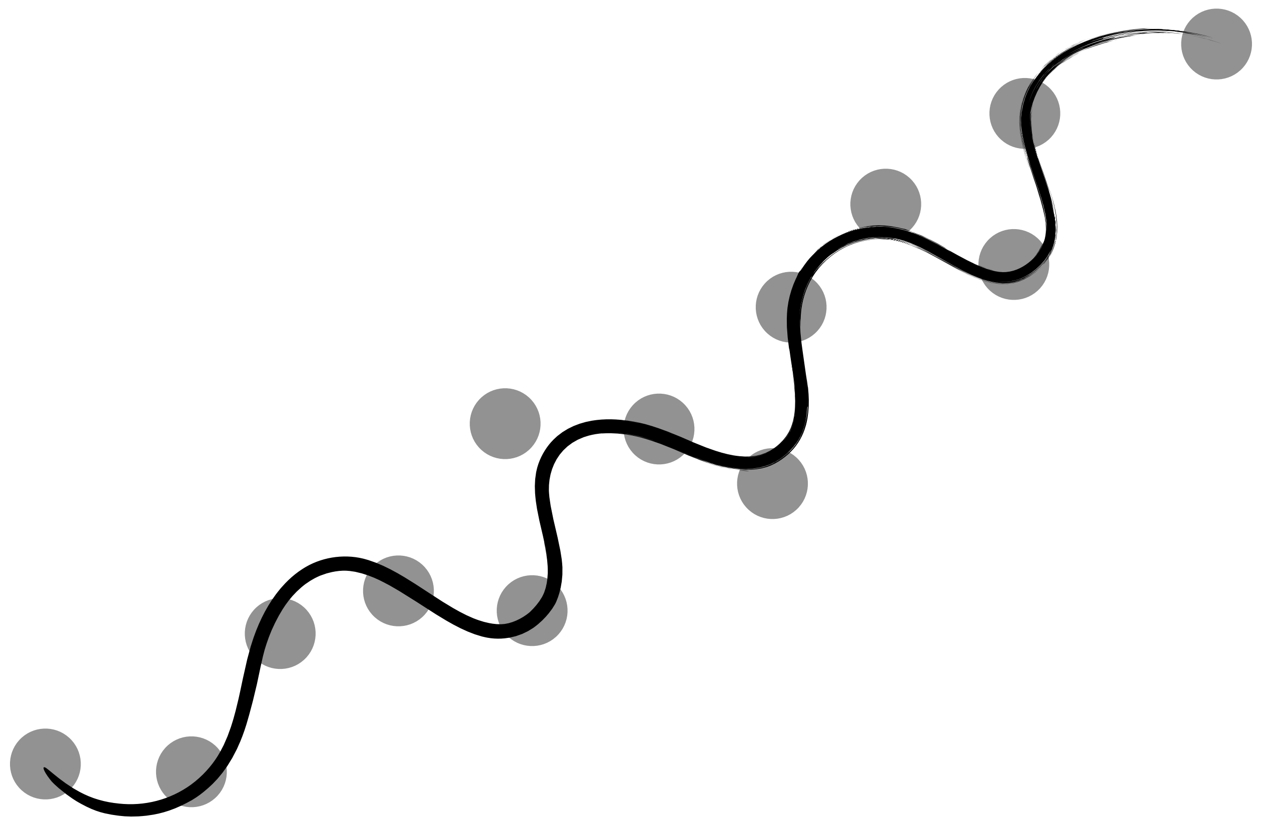
The **hypothesis space** H definies the set of functions an ML algorithm can find.

Examples:



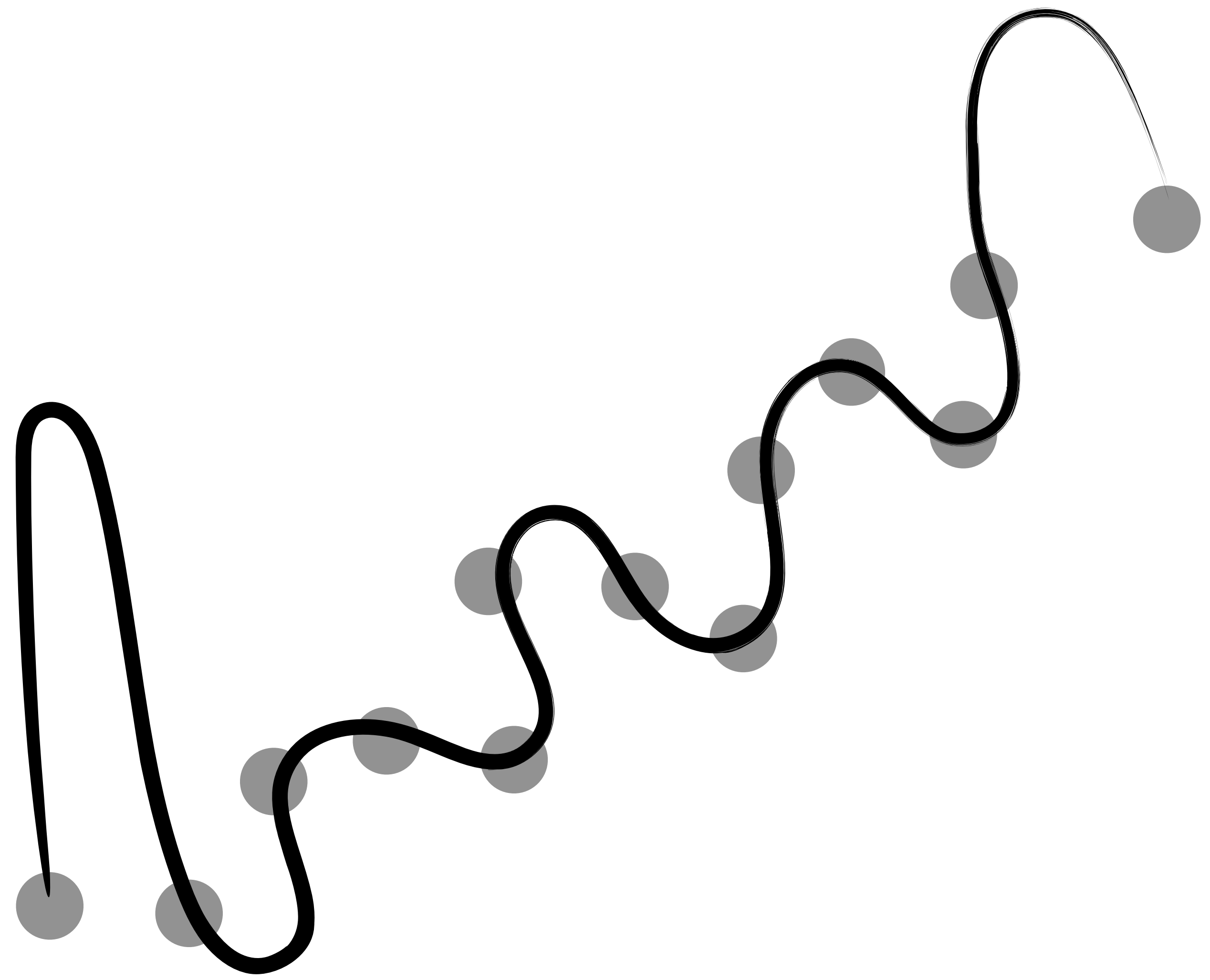
Linear

$$h(x) = w_1 x + w_0$$



Sinusoidal

$$h(x) = w_1 x + sin(w_0 x)$$



Degree-12 Polynomial

$$h(x) = \sum_{i=0}^{12} w_i x^i$$

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Loss function

The **loss function** L evaluates a function $h \in H$ with the dataset $D = \{(\mathbf{x}^{(1)}, y^{(1)}), \ldots, (\mathbf{x}^{(m)}, y^{(m)})\}$:

- Measures how far the predictions $h(\mathbf{x}^{(\mathbf{i})})$ are from labels y_i of examples $(\mathbf{x}^{(i)}, y^{(i)}) \in D$;
- Loss values $L(h)$ are always positive;
- The lower the $L(h)$, the better the function h – a function with loss $L(h) = 0$ (zero) correctly predicts the labels of all examples in D;
- Typically, loss functions are normalized to be independent from the size m of the dataset D.

Examples:

Zero-One Loss

$$L(h) = \frac{1}{m}\sum_{i=1}^{n} \delta_{h(\mathbf{x}^{(i)}) \neq y^{(i)}} \text{ where } \delta_{h(\mathbf{x}^{(i)}) \neq y^{(i)}} = \begin{cases} 1, & \text{if } h(\mathbf{x}^{(i)}) \neq y^{(i)} \\ 0, & \text{otherwise} \end{cases}$$

Mean Squared Error (MSE)

$$L(h) = \frac{1}{m}\sum_{i=1}^{n} (h(\mathbf{x}^{(i)}) - y^{(i)})^2$$

Mean Absolute Error (MAE)

$$L(h) = \frac{1}{m}\sum_{i=1}^{n} |\, h(\mathbf{x}^{(i)}) - y^{(i)} \,|$$

Evaluating Model's Performance

Given a dataset D, a hypothesys space H and a loss function L, we want to find the function $h \in H$ that:

$$h = argmin_{h \in H} L(h)$$

If we find a function $h \in H$ with low loss in D, how do we know that it also has loss low in new examples $(\mathbf{x}', y') \notin D$?

Consider the following "memorizer" function:

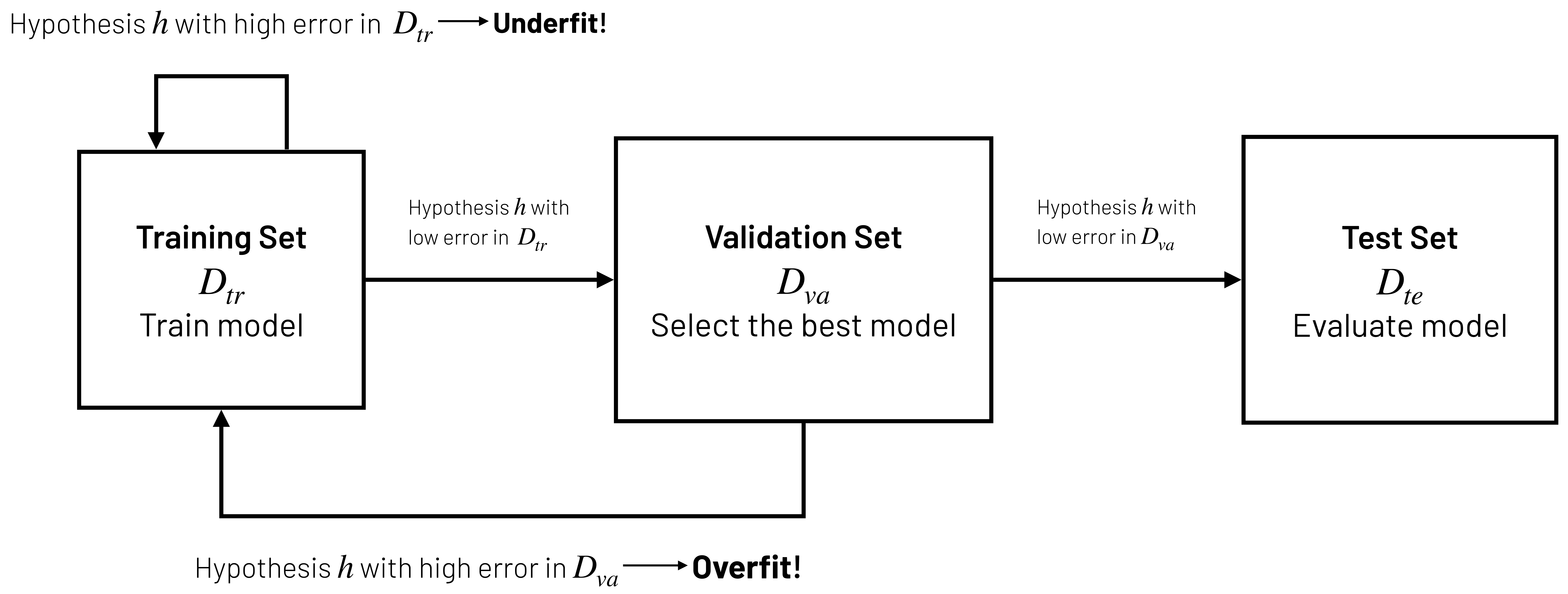
$$h(\mathbf{x}) = \begin{cases} y^{(i)}, & \text{if } \exists (\mathbf{x}^{(i)}, y^{(i)}) \in D, \text{such that}, \mathbf{x} = \mathbf{x}^{(i)} \\ 0, & \text{otherwise} \end{cases}$$

- Loss 0 for examples in D;
- Very high loss for unseen examples!

This problem is called overfit!

Evaluating Model's Performance

To evaluate a model on unseen examples, we typically divide the dataset D in 3 disjoint subsets: D_{tr}, D_{va} e D_{te}



Supervised Learning Algorithms

There are many supervised learning algorithsm and each oen assumes a different *hypothesis space* H:

- Linear Regression
- Logistic Regression
- Decision Trees
- K-Nearest Neighbors (KNN)
- Naive Bayes
- Suport Vector Machines (SVMs)
- Neural Networks

Next Lecture

L3: Linear Models

Discuss simple models that are the basis to how neural networks solve supervised learning problems:

- Linear Regression
- Perceptron
- Logistic Regression
- Gradient Descent